# UNITS AND MEASUREMENT

**Physical quantities, Unit, Classification of Units Systems, Basic Units of Systems, Supplementary Units, Dimensions of Physical Quantities, Dimensional Formula, To obtained Dimensional Formula of Different Quantities, Principle of Homogeneity of Dimension, Uses of Dimensional Equations, Conversion of One System of Units into another, Defects of Dimensional Analysis, To Check the Accuracy of a Formula , To Derive the Formula by Dimensional Analysis Method, Least Count, Precision of a Measurement, Accuracy of a Measurement, Order of Magnitude, Rounding Off, Percentage Error, Combination of Errors**

## PHYSICAL QUANTITIES

Those quantities which can describe the laws of physics & possible to measure are called physical quantities. A physical quantity is that which can be measured.

Physical quantity is completely specified :

If It has

- Numerical value only (ratio); e.g. refractive index dielectric constant etc.
- Magnitude only (scalar); e.g. mass, charge etc.
- Magnitude and Direction (vector); e.g. Displacement, torque etc.

**Note:** (1) There are also some physical quantities which are not completely specified even by magnitude, unit and Direction. These physical quantities are called tensors. Example moment of Inertia.

(2) Physical quantity = Numerical value x unit

Types of Physical Quantities : These are of two types

**(a) Fundamental quantities :** Those physical quantities which do not depend upon any other quantity are defined as fundamental quantities.

There are of seven fundamental quantities in SI system -

**(i)** Mass **(ii)** Length **(iii)** Time **(iv)** Temperature

**(v)** Electric current **(vi)** Luminous intensity **(vii)** Amount of substance

These quantities are also called **base quantities.**

**(b) Derived quantities :** The physical quantities which are derived from fundamental quantities and which depend upon them are defined as derived quantities.

For example speed $= \frac{\text{distance}}{\text{time}}$, Density $= \frac{\text{mass}}{\text{volume}}$

## SOLVED EXAMPLES

**Example.** Which of the following sets can not enter into the list of fundamental quantities in any system of units.

(1) Length, mass and velocity (2) Length, time and velocity

(3) Mass, time and velocity (4) Length, time and mass

**Solution.** The group of fundamental quantities are those quantities which do not depend upon other physical quantities in the group. But is set (2) we can predict the relation between given quantities as length = velocity × time. Hence set (2) can not enter in to the list of fundamental quantities.

Hence correct answer is (2)

# EXERCISE

**Question.** Find the dimensional formula of the following

(a) Spring constant (b) Surface Energy (c) Stress.

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## UNITS

The unit of a physical quantity is the reference standard used to measure it.

For the measurement of a physical quantity a definite magnitude of quantity is taken as standard and the name given to this standard is called unit.

Types of units

(i) **Fundamental units** (ii) **Derived units** (iii) **Practical units**

**(i) Fundamental units :** The units which are independent and which cannot be derived from other units, are defined as fundamental units, e.g. the units of mass, length and time.

**(ii) Derived units :** The units which are derived from the fundamental units are defined as derived units, e.q. units of velocity, momentum, work force etc.

## CLASSIFICATION OF UNIT SYSTEMS

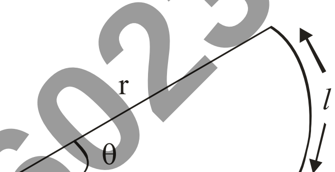
1. CGS
2. F.P.S (British System)
3. M.K.S (Metric System)
4. S.I. (System international)

## BASIC UNITS OF SYSTEMS

| Physics Quantity | C.G.S. | F.P.S. | M.K.S. | S.I. (symbol) |
|---|---|---|---|---|
| Mass | gram | pounds | kilogram | kilogram (kg) |
| Length | centimeter | foot | metre | metre (m) |
| Time | second | second | second | second (s) |
| Temperature | | | | kelvin (k) |
| Electric Current | | | | ampere (a) |
| Luminous Intensity | | | | candela (cd) |
| Amount of Substance | | | | mole (mol) |

## SUPPLEMENTARY UNITS

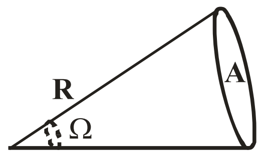
**(a) Radian :** 1 radian is the angle subtended by an arc of length equal to the radius, of the centre of the circle. if $l$ = r ⇒ θ - 1 radian




$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{l}{r}$$

**(b) Steradian :** It is defined as the solid angle subtended at the centre of a sphere by an area of its surface equal to the square of radius of the sphere.

Solid angle $\Omega = \frac{A}{R^2}$ if $A = R^2$, then $\Omega = 1$ steradian




## CHANGING UNITS

If we know the relations between the units of the same physical quantities in different systems we can change the unit of the required physical quantity by the same rules as we do in case of algebraic variables and numbers.

## SOLVED EXAMPLES

**Example 1:** The bulk modulus of a gas is 7 x $10^5$ dyne/cm$^2$. What is its value in SI units?

**Solution :** The relation between units of force and length in the two systems of units (CGS and SI) are

$1\text{ N} = 10^5\text{ dyne}$

$1\text{ dyne} = \frac{1}{10}\text{ N} = 10^{-5}\text{ N}$

$1\text{ m} = 10^2\text{ cm}$

or, $1\text{ cm} = \frac{1}{10^2}\text{ m} = 10^{-2}\text{ m}$

Substituting for dyne and cm in the value 7 x $10^5$ dyne/cm$^2$ ,

We get $7 \text{ x } 10^5 \times \frac{10^{-5} N}{10^{-2} m^2} = 7 \times 10^4 \frac{N}{m^2}$

**Example 2:** The speed of light in vaccum is 3 x $10^8$ m/s, what is its value in light year per year ?

**Solution :** Here we should know the relations between ly and m; s and y, i.e.

$1\ ly = 9.46 \text{ x } 10^{15}\text{m}$

$\therefore 1\text{ m} = \frac{1}{3.16 \times 10^7}\ ly$ & $1\ ly = 3.46 \text{ x } 10^7\text{ s}$

$\therefore 1\text{s} = \frac{1}{9.46 \times 10^{15}}\text{ y}$

Substituting c = 3 x $10^8$ m/s, we get

$c = 3 \times 10^8 \frac{\text{ly}}{9.46 \times 10^{15}} \times \frac{3.16 \times 10^7}{\text{ly}} \approx 1\ \ell\text{y / y}$

## EXERCISE

**Question 1.** The value of Planck constant 'h' is $6.63 \times 10^{-34}$ Js. What would be its numerical value in C.G.S. unit ?

**Question 2.** Find the value of impulse of 500 dyne second in a system based on meter, kilogram and minute as fundamental units.

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## DIMENSIONS OF PHYSICAL QUANTITIES

The limit of a derived quantity in terms of necessary basic units is called dimensional formula and the raised powers on the basic units are dimensions.

The basic units are represented as :

| | | | |
|---|---|---|---|
| Kilogram = M | Meter = L | Second = T | Kelvin = K |
| Ampere = A | Candela = Cd | Mole= mol. | |

**Note :**

**1.** A physical quantity may have a number of units but their dimensions would be same,

**Example** The units of velocity are: cms-1, ms-l, km S-I. But the dimensional formula is $M^0L^1T^1$.

**2.** Dimension does not depend on the unit of quantity.

## DIMENSIONAL FORMULA OR EQUATION

When a dimensional formula is equated to its physical quantity then the equation is called Dimensional, Equation. The dimensional equation can be obtained from the equation representing the relation between the physical quantities.

Dimension formula $= [M^a L^b T^c \theta^d]$ where a, b, c & d are the dimensions of M, L, T & θ respectively.

**Special Note -**

**1.** Pure number & pure ratio are dimension less.

Example 1, 2, π:, $e^x$, logx, sinθ, cos θ etc. & refractive indExample

**2.** Dimension less quantity may have unit.

Example Angle and solid angle.

**3.** The method of dimensions can not be applied to derive the formula if a physical quantity depends on more than three physical quantities.

**4.** Dimension less quantity may have unit. But unitless quantities are dimensionless.

Example angle - dimensionless but it has unit radian.

## SOLVED EXAMPLES

**Example 1.** Find the dimension of force?

**Solution.** By F = ma

⇒ Dimension Equation of $F = [M^1] [L^1 T^{-2}] = [MLT^{-2}]$

**Example 2.** Calculate the dimensional equation of Energy?

**Solution.** By E = W = Force × Displacement

Dimensional equation of 'E' $= [M^1 L^2 T^{-2}] [L^1] = [M^1 L^2 T^{-2}]$

# EXERCISE

**Question 1.** Find the dimensions of torque, impulse and angular momentum?

**Question 2.** Calculate the dimensions of kinetic energy and pressure?

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## TO OBTAINED DIMENSIONAL FORMULA OF DIFFERENT QUANTITIES

**Procedure:**

(i) Write the formula

(ii) Change derived units in the fundamental units.

(iii) Solve the equation except given quantity.

## UNITS AND DIMENSIONAL FORMULAE OF SOME PHYSICAL QUANTITIES OF INTEREST

| S.No. | Quantity | (Symbol) | Definition or formula | Dimensional formulae | C.G.S.Unit | SI |
|---|---|---|---|---|---|---|
| | **Unit Mechanics** | | | | | |
| 1. | Distance or \|Displacement\| | (S) | S or \|S\| | $[M^0L^1T^0]$ | cm | m |
| 2. | Speed or Velocity | (v) | $V = \frac{S}{t}$ | $[M^0L^1T^{-1}]$ | $cms^{-1}$ | $ms^{-1}$ |
| 3. | Accelaration | (a) | $a = \frac{v-u}{t}$ | $[M^0L^1T^{-2}]$ | $cms^{-2}$ | $ms^{-2}$ |
| 4. | Force | (F) | F = ma | $[M^1L^1T^{-2}]$ | dyne | newton(N) |
| 5. | Impulse | (I) | I = Ft | $[M^1L^1T^{-1}]$ | dyne sec or g $cms^{-1}$ | Ns or Kg $ms^{-1}$ |
| 6. | Momentum | (p) | p = mv | $[M^1L^1T^{-1}]$ | g $cms^{-1}$ | kg $ms^{-1}$ |
| 7. | Work | (W) | W =FS | $[M^1L^2T^{-2}]$ | erg | joule (J) |
| 8. | Power | (P) | P = W/t | $[M^1L^2T^{-3}]$ | erg $s^{-1}$ | $Js^{-1}$ or watt (W) |
| 9. | Energy(Kinetic or Potential | | | E = ½ $mv^2$ or mgh | $[M^1L^2T^{-2}]$ | erg joule |
| | or Heat or Atomic etc.)(E) | | or msθ or $mc^2$ | | | |
| 10. | Angle or | (θ) | $\theta = \frac{L}{r}$ | Dimensionless | degree | radian (rad) |
| | angular displacement | | | $[M^0L^0T^0]$ | | |
| 11. | Angular speed | (ω) | $\omega = \frac{\theta}{t}$ | $[M^0L^0T^{-1}]$ | deg $s^{-1}$ | rad $s^{-1}$ |
| 12. | Frequency | (υ) | $v = \frac{1}{t}$ | $[M^0L^0T^{-1}]$ | cycle/s | Hertz (Hz) |
| 13. | Angular acceleration | (α) | $\alpha = \frac{\omega - \omega_0}{t}$ | $[M^0L^0T^{-2}]$ | deg $s^{-2}$ | $rads^{-}$ |
| 14. | Torque | (τ) | τ = Fd | $[ML^2T^{-2}]$ | dyne cm | Nm |
| 15. | Angular momentum | (L) | L = mvr or h = $\underline{E}$ | $[ML^2T^{-1}]$ | erg s | Js |

| | | | | | |
|---|---|---|---|---|---|
| | Or Planck's constant (h) | $\upsilon$ | | | |
| 16. | Moment of inertia (I) | $I = mr^2$ | $[ML^2T^0]$ | $gcm^2$ | $kg\,m^2$ |
| 17. | Radius of gyration (k) | $k = \frac{\sqrt{Moment\ of\ inertia}}{\sqrt{mass}}$ | $[ML^2T^2]$ | | |
| 18. | Force constant or (k) stiffness or spring constant | $k = \frac{F}{x}$ | $[M^1L^0T^{-2}]$ | dyne/cm | N/m |
| 19. | Gravitational constant (G) | $G = \frac{Fr^2}{m_1m_2}$ | $[M^{-1}L^3T^{-2}]$ | dyne $cm^2/g^2$ | $Nm^2/kg^2$ |
| 20. | Gravitational field inensity(g) | $g = \frac{F}{}$ | $M^0L^1T^{-2}$ | dyne/g or $cm/s^2$ | N/kg or $m/s^2$ |
| 21. | Gravitational potential ($V_g$) | $V_g = \frac{U}{m}$ | $[M^0L^2T^{-2}]$ | erg/g | J/kg |
| 22. | Stress | S = restoring force/area | $[M^1L^{-1}T^{-2}]$ | $dyne/cm^2$ | $N/m^2$ |
| 23. | Strain | $\varepsilon = \frac{\text{Change in dim}^n}{\text{Original dim}^n}$ | $[M^0L^0T^0]$ (Dimensionless) | no unit | no unit |
| 24. | Modulus or coefficient of elasticity (Young's, Bulk) or Modulus of rigidity (E) | $E = \frac{stress}{Strain}$ | $[M^1L^{-1}T^{-2}]$ | $dyne/cm^2$ | $N/m^2$ |
| 25. | Poisson's ratio ($\sigma$) | $\sigma = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$ | Dimensionless $[M^0L^0T^0]$ | no unit | no unit |
| 26. | Surface tension (T) | $T = \frac{f}{l}$ | $[M^1L^0T^{-2}]$ | dyne/cm | N/m |
| 27. | Surface energy (S) | $S = \frac{w}{\Delta A}$ | $[M^1L^0T^{-2}]$ | $erg/cm^2$ | $J/m^2$ |
| 28. | Velocity gradient | $\frac{dv}{dx}$ | $[M^0L^0T^{-1}]$ | $s^{-1}$ | $s^{-1}$ |
| 29. | Cofficient of viscosity ($\eta$) | $\eta = \frac{F}{A}\frac{(dv)}{(dx)}$ | $[ML^{-1}T^{-1}]$ | $gcm^{-1}s^{-1}$ or Poise or dyne s $cm^{-2}$ | $kgm^{-1}s^{-1}$ or decapoise or Ns $m^{-2}$ |

**HEAT**

| | | | | | |
|---|---|---|---|---|---|
| 30. | Mechanical Equivalent of heat (J) | $J = \frac{W}{H}$ | $[M^0L^0T^0]$ (Dimensionless) | erg/calorie | J/Calorie |
| 31. | Specific heat (s) | $s = \frac{Q}{m\Delta T}$ | $[M^0L^2T^{-2}\theta^{-1}]$ | erg g-1 $C^{o-1}$ | J $kg^{-1}K^{-1}$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| 32. | Latent heat of fusion or vaporisation | (L) | $L = \frac{Q}{m}$ | $[M^0L^2T^{-2}]$ | erg $g^{-1}$ | J $kg^{-1}$ |
| 33. | Entropy | (S) | $S = \frac{Q}{T}$ | $[M^1L^2T^{-2}\theta^{-1}]$ | erg /°C | J/K |
| 34. | Gas constant | (R) | $R = \frac{PV}{\mu T}$ | $[M^1L^2T^{-2}\theta^{-1}]mol^{-1}$ | erg $mol^{-1}\,°C^{-1}$ | J $mol^{-1}K^{-1}$ |
| 35. | Boltzmann constant | (k) | $k = \frac{R}{N_A}$ | $[M^1L^2T^{-2}\theta^{-1}]$ | erg $°C^{-1}$ | J $K^{-1}$ |
| 36. | Mean free path | (λ) | $\lambda = \frac{1}{\sqrt{2\pi d^2 n}}$ | $[M^0L^1T^{-0}]$ | cm | m |
| 37. | Molar specific heat | (C) | $C = \frac{Q}{\mu \Delta T}$ | $[M^1L^2T^{-2}\theta^{-1}]$ | erg $mol^{-1}°C^{-1}$ | J $mol^{-1}K^{-1}$ |
| 38. | Temperature gradient | | $\frac{d\theta}{dx}$ | $[M^0L^{-1}T^0\theta^{1}]$ | °C/cm | K/m |
| 39. | Cofficient of thermal conductivity | | $K = \frac{Q}{A\frac{d\theta}{dx}}$ | $[M^1L^1T^{-3}\theta^{-1}]$ | erg $cm^{-1}s^{-1}°C^{-1}$ | J $m^{-1}s^{-1}K^{-1}$ or w $m^{-1}K^{-1}$ |
| 40. | Thermal resistance | $(R_H)$ | $R_H = \frac{\Delta\theta . t}{Q}$ | $[M^{-1}L^{-2}T^3\,\theta^{-1}]$ | °C s/erg | Ks/Jit |
| 41. | Intensity of radiation | (E) | $E = \frac{Q}{At}$ | $[M^1L^0T^{-3}]$ | erg $cm^{-2}s^{-1}$Jm | $^{-2}$ $s^{-1}$ or $wm^{-2}$ |
| 42. | Stefan's constant | (σ) | $\sigma = \frac{E}{T^4}$ | $[M^1L^0T^{-3}\,\theta^{-4}]$ | erg $s^{-1}$ $cm^{-2}$ $°C^{-4}$ | $Js^{-1}$ $m^{-2}$ $K^{-4}$ or w $m^{-2}$ $K^{-4}$ |
| 43. | Wein's constant | (b) | $b = \lambda_m T$ | $[M^0L^1T^0\,\theta^1]$ | cm°C | mk |
| 44. | Emissivity | (ε) | $\varepsilon = \frac{Q_{grey}}{Q_{black}}$ | Dimensionless $[M^0L^0T^0]$ | no units | no units |
| 45. | Rate of cooling | | $\frac{d\theta}{dt}$ | $\theta T^{-1}$ | $°Cs^{-1}$ | $Ks^{-1}$ |

**OSCILLATIONS AND WAVES**

| | | | | | | |
|---|---|---|---|---|---|---|
| 46. | Amplitude of wave | (a) | maximum displacement from mean position | $[M^0L^1T^0]$ | cm | m |
| 47. | Wavelength | ($\lambda$) | Distance between two successive crests | $[M^0L^1T^0]$ | cm | m |
| 48. | Frequency | ($\nu$) | oscillations per second | $[M^0L^0T^{-1}]$ | Hertz | Hertz |
| 49. | Angular Frequency | ($\omega$) | $\omega = 2\pi\nu$ | $[M^0L^0T^{-1}]$ | Hertz | Hertz |
| 50. | Wave constant | (k) | $k = 2\pi / \lambda$ | $[M^0L^{-1}T^0]$ | $cm^{-1}$ | $m^{-1}$ |
| 51. | Wave Number | ($\bar{\nu}$) | $\bar{\nu} = 1/\lambda$ | $[M^0L^{-1}T^0]$ | $cm^{-1}$ | $m^{-1}$ |
| 52. | Intensity of wave | (I) | $I = 2\pi^2 \upsilon^2 a^2 \rho$ | $[M^1L^0T^{-3}]$ | erg $cm^{-2}$ $s^{-1}$ | $Wm^{-2}$ |
| 53. | Energy density | (u) | u = Energy /vol. | $[ML^{-1}T^{-2}]$ | erg/$cm^3$ | J/$m^3$ |
| 54. | Wave velocity | (v) | $v = \upsilon\lambda$ | $[M^0L^1T^{-1}]$ | $cms^{-1}$ | $ms^{-1}$ |

**ELECTRICITY AND MAGNETISM**

| | | | | | | |
|---|---|---|---|---|---|---|
| 55. | Electric charge | (q) | q = It | $M^0L^0T^1 I^1$ | Stat. coulomb | Coulomb (C) |
| 56. | Permitivity constant | ($\varepsilon$ or $\varepsilon_o$) | $\varepsilon = \dfrac{q_1q_2}{F.4\pi r^2}$ | $[M^{-1}L^{-3}T^{-4}I^2]$ | $\dfrac{stat\ coulomb}{dyne\ cm^2}$ | $\dfrac{C^2}{Nm^2}$ |
| 57. | Relative permitivity ($\varepsilon_r$) Or dielectric constant (K) | | $\varepsilon_r$ or $K = \dfrac{\varepsilon}{\varepsilon_0}$ | Dimensionless | no units | no units |
| 58. | Electric potential | (V) | $V = \dfrac{W}{Q}$ | $[M^1L^2T^{-3}I^{-1}]$ | erg/ stat coulomb or stat volt | J/C Or Volt (V) |
| 59. | Potential gradient | | $\dfrac{dV}{dx}$ | $[M^1L^1T^{-3}I^{-1}]$ | Stat volt/cm | V/m |
| 60. | Electric intensity | (E) | $E = \dfrac{F}{Q}$ | $[M^1L^1T^{-3}I^{-1}]$ | dyne/ stat coulomb | N/C |
| 61. | Electric flux | ($\varphi_E$) | $\varphi_E = EA$ | `$[M^1L^3T^{-3}I^{-1}]$ | stat volt cmvo | lt meter(Vm) |
| 62. | Electric dipole moment ($P_E$) | | $P_E = qa$ | $[M^0L^1T^1I^1]$ | stat coulomb cm | C m |
| 63. | Electric susceptibility ($\chi$) | | $\chi = K-1$ | Dimensionless | no units | no units |
| 64. | Capacitance | (C) | $C = \dfrac{Q}{V}$ | $[M^{-1}L^{-2}T^4I^2]$ | stat farad | Farad (F) |
| 61. | Resistance | (R) | $R = \dfrac{V}{I}$ | $[M^1L^2T^{-3}I^{-2}]$ | stat ohm | Ohm ($\Omega$) |

| | | | | | | |
|---|---|---|---|---|---|---|
| 62. | Specific resistance Or Resistivity | ($\rho$) | $\rho = \frac{RA}{l}$ | $[M^1L^3T^{-3}I^{-2}]$ | stat ohm cm | $\Omega$m |
| 63. | Conductance | (c) | $c = \frac{1}{R}$ | $[M^{-1}L^{-2}T^3I^2]$ | stat mho or | mho or Sieman |
| 64. | Conductivity | ($\sigma$) | $\sigma = \frac{1}{\rho}$ | $[M^{-1}L^{-3}T^3I^2]$ | $(\text{stat ohm})^{-1}\text{cm}^{-1}$ | ohm$^{-1}$m$^{-1}$ or of resistance |
| 64. | Temperature coefficient ($\alpha$) | | $\alpha = \frac{R_2 - R_1}{R_1 t}$ | $[M^\circ L^\circ T^\circ \theta^{-1}]$ | $^\circ C^{-1}$ | $K^{-1}$ |
| 65. | Magnetic Field | (B) | $B = F/IL$ | $[M^0T^{-2}I^{-1}]$ | gauss | Tesla |
| 66. | Magnetic flux | ($\phi$) | $\phi = B.A$ | $[M^0L^2T^{-2}I^{-1}]$ | gauss.cm$^2$ | Weber |
| 67. | Permeability | ($\mu$) | $\mu = \frac{B.2a}{I}$ | $[M^0L^1T^{-2}I^{-2}]$ | - | TmA$^{-1}$ |
| 68. | Magnetic Dipole moment | (m) | $m = NIA$ | $[M^0L^2T^0I^1]$ | - | Am$^2$ |
| 69. | Self or Mutual Inductance(L) | | $L = \frac{\phi}{I}$ | $[M^0L^2T^{-2}I^{-2}]$ | - | Henry |
| 70. | Time Constant (RC-Ckt) | | $\tau = RC$ | $[M^0L^0T^1]$ | s | s |
| 71. | Time constant (LR-Ckt) | | $\tau = L/R$ | $[M^0L^0T^1]$ | s | s |
| 72. | Planck's Constant | (h) | $h = E/\nu$ | $[M^1L^2T^{-1}]$ | erg.s | Js |

## PRINCIPLE OF HOMOGENEITY OF DIMENSION

The dimension of physical quantity on the left hand side of dimensional equation should equal to the net dimensions of all physical quantities on the right hand side of it.

## SOLVED EXAMPLES

**Example 1.** Check the dimensional validity of the euqation $s = ut + \frac{1}{2}gt^2$

**Solution.** $[L] = [L.T^{-1}.T] + [L.T^{-2}.T^2]$

$[L] = [L] + [L]$

**Example 2.** In the formula; $N = -D\left[\frac{n_2 - n_1}{x_2 - x_1}\right]$, D = diffusion coefficient, $n_1$ and $n_2$ is number of molecules in unit volume along $x_1$ and $x_2$. Which represents distances where N is number of molecules passing through per unit area per unit time calculate dimensional equation of D.

**Solution.** By Homogeneity theory of Dimension

$$\text{Dimension of (N)} = \text{Dimension of D} \times \frac{\text{Dimension of }(n_2 - n_1)}{\text{Dimesion of }(x_2 - x_1)}$$

$$\frac{1}{L^2T} = \text{Dimension of D} \times \frac{L^{-3}}{L}$$

$$\Rightarrow \text{Dimension of 'D'} = \frac{L}{L^{-3} \times L^2T} = \frac{L^2}{T} = L^2T^{-1}$$

# EXERCISE

**Question 1.** The position x of a particle at time t is given by $x = \frac{v_o}{a}(1 - e^{-at})$ where $v_0$ is a constant and a>0. The dimensions of $v_0$ and a are

(a) $M^0LT^{-1}$ & $T^{-1}$ (b) $M^0LT^0$ & $T^{-1}$

(c) $M^0LT^{-1}$ & $LT^{-2}$ (d) $M^0LT^{-1}$ & T

**Question 2.** If in the formula $x = 3yz^2$, x and z represent electrical capacitances and magnetic induction then calculate dimensional equation of y?

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## USES OF DIMENSIONAL EQUATIONS

Following are the uses of dimensional equations :

1. Conversion of one system of units in to another.
2. Checking the accuracy of various formula or equation.
3. Derivation of formula.

## CONVERSION OF ONE SYSTEM OF UNITS INTO ANOTHER

Let the numerical values are $n_1$ and $n_2$ of a given quantity Q in two unit system and the units are-

$U_1 = M_1^aL_1^bT_1^c$ and $U_2 = M_2^aL_2^bT_2^c$ (in two systems respectively)

Therefore, By the principle nU = constant

$\Rightarrow \; n_2U_2 = n_1U_1$

$$n_2[M_2^a\,L_2^b\,T_2^c] = n_1[M_1^a\,L_1^b\,T_1^c]$$

$$\Rightarrow \; n_2 = \frac{n_1[M_1^a\,L_1^b\,T_1^c]}{[M_2^a\,L_2^b\,T_2^c]} \quad \Rightarrow \quad \boxed{n_2 = \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c n_1}$$

## DEFECTS OF DIMENSIONAL ANALYSIS

(a) While deriving a formula the proportionally constant cannot be found.

(b) The formula for a physical quantity depending on more than three other physical quantities cannot be derived. It can be checked only.

(c) The equations of the type v = u ± at cannot be derived. They can be checked only

(d) The equations containing trigonometrical functions (sinθ, cosθ, etc), logarithmic functions (logx, $\log x^3$ etc) and exponential functions ($e^x$, $e^{x^2}$ etc) cannot be derived. They can be checked only.

## TO CHECK THE ACCURACY OF A FORMULA

It is based on homogeneity principle of dimension according to it, formula is correct when L.H.S. = R.H.S. dimensionally.

## SOLVED EXAMPLES

**Example 1.** Test the correctness of the formula $T = 2\pi\sqrt{\frac{\ell}{g}}$ , where , T = time period $\ell$ = length of pendulum and g = Acc. due to gravity.

**Solution.** In L.H.S Dimension equation of T $\Rightarrow M^0L^0T^1$

In R.H.S. Dimension equation of ; $2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \left[\frac{M^0 L^1 T^0}{L^1 T^{-2}}\right]^{1/2}$ = $[M^0L^0T^2]^{1/2}$ = $M^0L^0T^1$

$\because$ L.H.S. = R.H.S. Dimensionally. Therefore, the given formula is correct.

## EXERCISE

**Question 1.** Check the equations given below, whether these are dimensionally correct or not?

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 = u^2 + 2as$ (iv) $s_n = u + \frac{1}{2}a(2n-1)$

**Question 2.** Find the dimensions of a'a & 'b' in equation $\left(p + \frac{a}{v^2}\right)(v-b) = RT$ , where symbols have their usual meanings.

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## TO DERIVE THE FORMULA BY DIMENSIONAL ANALYSIS METHOD

Let a physical quantity x depends on the another quantities P, Q and R.

Then $x \propto (P)^a (Q)^b (R)^c$

$x = k (P)^a (Q)^b (R)^c$ ...........(1)

Now consider dimensional formula of each quantity in both side -

$$M^xL^yT^z = [M^{x_1}L^{y_1}T^{z_1}]^a \ [M^{x_2}L^{y_2}T^{z_2}]^b \ [M^{x_3}L^{y_3}T^{z_3}]^c$$

$$\Rightarrow \quad M^xL^yT^z = M^{ax_1}L^{ay_1}T^{az_1} \ M^{bx_2}L^{by_2}T^{bz_2} \ M^{cx_3}L^{cy_3}T^{cz_3}$$

$$\Rightarrow \quad M^xL^yT^z = M^{ax_1+bx_2+cx_3}L^{ay_1+by_2+cy_3}T^{az_1+bz_2+cz_3}$$

Now comparing the powers of both sides -

$ax_1 + bx_2 + cx_3 = x$ ...........(2)

$ay_1 + by_2 + cy_3 = y$ ...........(3)

$az_1 + bz_2 + cz_3 = z$ ...........(4)

After solving equation (2), (3) and (4) value of a, b and c will be m, n and o may be find out

Now substitute the values of x, y and z in equation (1)

Then obtained formula will be- $x = (P)^m (Q)^n (R)^o$

## SOLVED EXAMPLES

**Example .** The time of oscillation (T) depends upon the density 'd' radius 'r' and surface Tension. Obtain the formula for T by dimensional method.

**Solution.** $T \propto (d)^a (r)^b (s)^c \quad \Rightarrow \quad T = k (d)^a (r)^b (s)^c$ ...(1)

Taking dimension of each quantity in both sides.

$M^0L^0T^1 = [M^1L^{-3}T^0]^a [L^1]^b [M^1L^3T^{-2}]^c$

$\Rightarrow \quad M^0L^0T^1 = [M^{a+c}] [L^{-3a+b}] [T^{-2c}]$

$[M^0L^0T^1] = [M^{a+c}L^{-3a+b}T^{-2c}]$

comparing the dimensions of both sides

$a + c = 0$ ... (2)

$-3a + b = 0$ ...(3)

$-2c = 1$ or $c = -1/2$ ...(4)

Substituting value of c in equation (3)

$a + (-1/2) = 0$

$\Rightarrow \quad a = 1/2$

Now putting $a = 1/2$ in equation (3)

$\Rightarrow \quad -3\left(\frac{1}{2}\right) + b = 0 \quad \Rightarrow \quad b = 3/2$

On substituting value a, b and c in equation (1) $T = k(d)^{1/2} (r)^{3/2} (s)^{-1/2}$

$\Rightarrow \quad T = \sqrt{\frac{dr^3}{s}}$

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## EXERCISE

**Question 1.** In a simple pendulum method if 'T' is time period of oscillatio, '$l$' is length of thread and 'g' is gravitational acceleration then construct a formula showing the realation between 'T', '$l$' & 'g', using the method of dimensions.

**Question 2.** The time period T of the oscillations of a large fluid star, oscillating under its own gravitational attraction, may depend on its mean radius R, its mean density ρ and the gravitation constant G, Using dimensional consider-tions, show that T is independent of R and is given by $T = \frac{k}{\sqrt{\rho G}}$ where k is a dimensionless constant.

**Question 3.** Assuming that the mass m of the largest stone that can be moved by a flowing river depends upon the velocity v of water, its density ρ and acceleration due to gravity g, show that m varies as the sixth power of the velocity of water in the river.

## SOME IMPORTANT PREFIXES

### S.I. PREFIXES

| S.No. | Perfix | Symbol | Power of 10 |
|---|---|---|---|
| 1 | exa | E | 18 |
| 2 | peta | P | 15 |
| 3 | tera | T | 12 |
| 4 | giga | G | 9 |
| 5 | mega | M | 6 |
| 6 | kilo | k | 3 |
| 7 | hector | h | 2 |
| 8 | deca | da | 1 |
| 9 | deci | d | –1 |
| 10 | centi | c | –2 |
| 11 | milli | m | –3 |
| 12 | micro | μ | –6 |
| 13 | nano | n | –9 |
| 14 | pico | p | –12 |
| 15 | femto | f | –15 |
| 16 | atto | a | –18 |

### SOME PRACTICAL UNITS OF LENGTH

1. Light year = $9.46 \times 10^{15}$ m
2. Parsec = $3.084 \times 10^{16}$m
3. Fermi = $10^{-15}$ m
4. Angstron (A) = $10^{-10}$ m
5. Micron/Micrometer = $10^{-6}$ m
6. Nano meter = $10^{-9}$ m
7. Picometer = $10^{-12}$ m
8. Acto meter = $10^{-18}$ m
9. Astronomical unit (A.U.) =$1.496 \times 10^{11}$ m
10. Otto meter = $10^{-21}$ m

## SOME IMPORTANT PRACTICAL UNITS

| S.No. | Quantity | Unit |
|---|---|---|
| 1. | Mass | Solar mass = 2 x $10^{30}$ |
| | | Dalton = $1.66 \times 10^{-27}$ kg |
| | | Chander Shekhar =1.4 times of mass of sun |
| 2. | Pressure | Pascal = 1 N/m$^2$ |
| | | Bar = $10^5$ N/m$^2$, Torr, mm of Hg coloumn |
| 3. | Area | barn = $10^{-28}$ m$^2$ |
| 4. | Radio Activity | Baquerrel |
| 5. | Cancer | Rontgen |
| 6. | Time | Shake = $10^{-8}$ sec |

## SOME USEFUL CONSEQUENCES

1. Quantities having identical dimensions

(a) Latent heat, Gravitational potential $[M^0L^2T^{-2}]$

(b) Planck constant , Angular momentum $[ML^2T^{-1}]$

(c) Rydberg constant, Propagation constant $[M^0L^{-1}T^0]$

(d) Surface tension, spring constant , Surface energy $[ML^0T^{-2}]$

(e) Frequency, Angular velocity, velocity gradient $[M^0L^0T^{-1}]$

(f) Wave length, Radius of gyration, Light year $[M^0L^1T^0]$

(g) Heat, work, energy, torque $[ML^2T^{-2}]$

(h) Pressure, stress, Modulii of elasticity, Energy density $[ML^{-1}T^{-2}]$

(i) Electric intensity / Magnetic induction, Velocity, $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

(j) Impulse, Momentum

(k) Strain, Refractive index, Relative permittivity, Specific gravity.

2. The dimensions of physical quantity do not depend upon system of units used to represent that quantity.
3. A rationalized system of units is that which uses only one unit for a base quantity e.g. M.K.S and SI.
4. A coherent system of units is that in which all derived units are obtained by multiplying or dividing the base units.
5. Greater the number of significant figures in a measurement , lower will be its percentage error.